



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

II. Solution by W. W. BEMAN. A. M., Professor of Mathematics, State University, Ann Arbor, Mich.

$$\left(\tan^{-1}x - \frac{x}{1+x^2}\right) \frac{d^2y}{dx^2} = \frac{2x}{(1+x^2)^2} \left(\frac{x}{dx} \frac{dy}{dx} - y\right).$$

Writing the equation in the form

$$\frac{d}{dx} \left( \frac{\tan^2 x}{x} \right) \cdot \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{1}{1+x^2} \right) \cdot \frac{d}{dx} \left( \frac{y}{x} \right)$$

it is obvious that  $y = \tan^{-1}x$  and  $y = x$  are independent particular integrals. Hence the complete primitive is  $y = c_1 \tan^{-1}x + c_2 x$ .

Solutions of this problem were also received from L. C. WALKER, and G. B. M. ZERR.

126. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Find the volume contained between the conical surface whose equation is  $z = a - \sqrt{(x^2 + y^2)}$ , and the planes whose equations are  $x = z$  and  $x = 0$  by the formula  $\int \int \int dx dy dz$ . [Todhunter's *Integral Calculus*.]

Solution by W. J. GREENSTREET. M. A., Editor of The Mathematical Gazette, Stroud, Gloucestershire, England.

The cone clearly extends from vertex  $(0, 0, a)$  towards  $z = 0$ . Hence in

$$\int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} dx dy dz$$

we have  $z_1 = x$ ;  $z_2 = a - \sqrt{(x^2 + y^2)}$ ;  $x = a - \sqrt{(x^2 + y^2)}$ .

$\therefore y^2 = a^2 - 2ax$ ,  $x_1 = 0$ , and for  $x_2$  we have  $y_1 = y_2$ .  $\therefore x = \frac{1}{2}a$ .

$$\therefore \text{Volume} = \int_0^{\frac{1}{2}a} \int_{y_1}^{y_2} [a - x - \sqrt{(x^2 + y^2)}] dx dy$$

$$= \int_0^{\frac{1}{2}a} \left[ (a-x)(y_2 - y_1) - \frac{y}{2} \sqrt{(x^2 + y^2)} - \frac{x^2}{2} \log y + \sqrt{(x^2 + y^2)} \right]_{y_1}^{y_2} dx$$

$$= \int_0^{\frac{1}{2}a} \left[ 2(a-x) \sqrt{(a^2 - 2xa)} - \sqrt{(a^2 - 2xa)(a-x)} - \frac{x^2}{2} \log \frac{a-x+\sqrt{(a^2-2ax)}}{a-x-\sqrt{(a^2-2ax)}} \right] dx.$$

Put  $2x = a \sin^2 \phi$ .

$$\therefore V = \int_0^{\frac{1}{2}\pi} a \sin \phi \cos \phi d\phi \left[ (1 + \cos \phi) \frac{a^2}{2} \cos \phi - \frac{a^2}{8} \sin^4 \phi \log \left( \frac{1 + \cos \phi}{1 - \cos \phi} \right)^2 \right]$$

$$= \frac{a^3}{2} \int_0^{\frac{1}{2}\pi} \left[ \sin \phi \cos^2 \phi + \sin \phi \cos^4 \phi - \sin^5 \phi \cos \phi \log \frac{1 + \cos \phi}{\sin \phi} \right] d\phi$$

$$\begin{aligned}
 &= \frac{a^3}{2} \left( \frac{1}{3} + \frac{1}{5} \right) - \left[ \frac{a^3 \sin^6 \phi}{12} \log \frac{1 + \cos \phi}{\sin \phi} \right]_0^{\frac{1}{2}\pi} - \frac{a^3}{12} \int_0^{\frac{1}{2}\pi} \sin^6 \phi \left( \frac{\cos \phi}{\sin \phi} + \frac{\sin \phi}{1 + \cos \phi} \right) dx \\
 &= \frac{4}{5} a^3 - \frac{1}{12} a^3 \int_0^{\frac{1}{2}\pi} [\cos \phi \sin^5 \phi + \sin^5 \phi (1 - \cos \phi)] d\phi = \frac{4}{5} a^3 - \frac{a^3}{12} \cdot \frac{4.2}{5.3} = \frac{2}{9} a^3.
 \end{aligned}$$

Similar solutions were received from **GEORGE LILLEY**, **LON C. WALKER**, **G. B. M. ZERR**, and **J. SCHEFFER**.

## MECHANICS.

127. Proposed by **F. P. MATZ**, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, Ohio.

Develop the Fourier Series to represent the temperature of a circular wire of uniform cross-section, in which the temperatures of the four quadrants are in order  $t, 2t, 3t, 4t$ .

Solution by **G. B. M. ZERR**, A.M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

From Fourier's *Analytical Theory of Heat*, page 217, we get for the complete solution of the linear and varied movement of heat in a ring after a time  $T$  the following:

$$\begin{aligned}
 v &= e^{-hT} \left[ \frac{1}{2\pi} \int_0^{2\pi} f(x) dx + \frac{1}{\pi} \sum_{n=1}^{n=\infty} e^{-n^2 k T} \sin nx \int_0^{\frac{1}{2}\pi} \sin nx f(x) dx \right. \\
 &\quad \left. + \frac{1}{\pi} \sum_{n=1}^{n=\infty} e^{-n^2 k T} \cos nx \int_0^{\frac{1}{2}\pi} \cos nx f(x) dx \right],
 \end{aligned}$$

$$\begin{aligned}
 \text{where } f(x) &= \frac{1}{2\pi} \int_0^{2\pi} f(x) dx + \frac{1}{\pi} \sum_{n=1}^{n=\infty} \sin nx \int_0^{2\pi} \sin nx f(x) dx \\
 &\quad + \frac{1}{\pi} \sum_{n=1}^{n=\infty} \cos nx \int_0^{2\pi} \cos nx f(x) dx.
 \end{aligned}$$

$$\int_0^{2\pi} f(x) dx = t \int_0^{\frac{1}{2}\pi} dx + 2t \int_{\frac{1}{2}\pi}^{\pi} dx + 3t \int_{\pi}^{\frac{3}{2}\pi} dx + 4t \int_{\frac{3}{2}\pi}^{2\pi} dx = 5\pi t.$$

$$\begin{aligned}
 \int_0^{2\pi} \sin nx f(x) dx &= t \int_0^{\frac{1}{2}\pi} \sin nx dx + 2t \int_{\frac{1}{2}\pi}^{\pi} \sin nx dx + 3t \int_{\pi}^{\frac{3}{2}\pi} \sin nx dx + 4t \int_{\frac{3}{2}\pi}^{2\pi} \sin nx dx \\
 &= \frac{t}{n} (\cos \frac{1}{2}\pi n + \cos \pi n + \cos \frac{3}{2}\pi n) - \frac{3t}{n} = -\frac{4t}{n}, \text{ except when } n=4m.
 \end{aligned}$$

$$\int_0^{2\pi} \cos nx f(x) dx = t \int_0^{\frac{1}{2}\pi} \cos nx dx + 2t \int_{\frac{1}{2}\pi}^{\pi} \cos nx dx + 3t \int_{\pi}^{\frac{3}{2}\pi} \cos nx dx + 4t \int_{\frac{3}{2}\pi}^{2\pi} \cos nx dx$$